Name________________________________

Activity - The Derivative as a Function

Part 1. Consider the function $f(x) = 4x - x^2$.

(a) Use the limit definition to compute the following derivative values: $f'(0)$, $f'(1)$, $f'(2)$, and $f'(3)$. No credit given if your work does not show the limit definition.

(b) Observe that the work to find $f'(a)$ is the same, regardless of the value of a. Based on your work in (a), what do you conjecture is the value of $f'(4)$? How about $f'(5)$? (Note: you should ***not*** use the limit definition of the derivative to find either value.)

(c) Conjecture a formula for $f'(a)$ that depends only on the value a. That is, in the same way that we have a formula for $f(x)$ (recall $f(x) = 4x - x^2$), see if you can use your work above to guess a formula for $f'(a)$ in terms of a.

Part 2. Use a separate sheet of paper for this part.

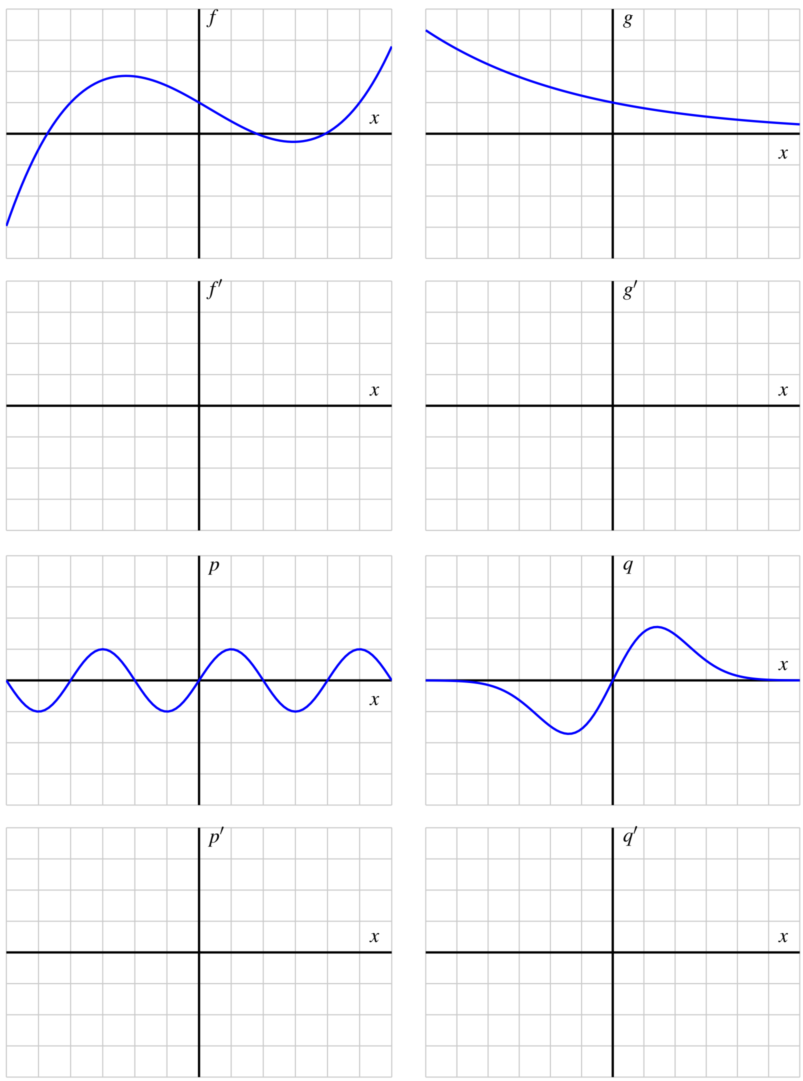
(a) Let g be the function given by the rule $g(x) = |x|$. Graph this function.
(b) Reasoning visually, explain why g is differentiable at every point x such that $x \neq 0$.

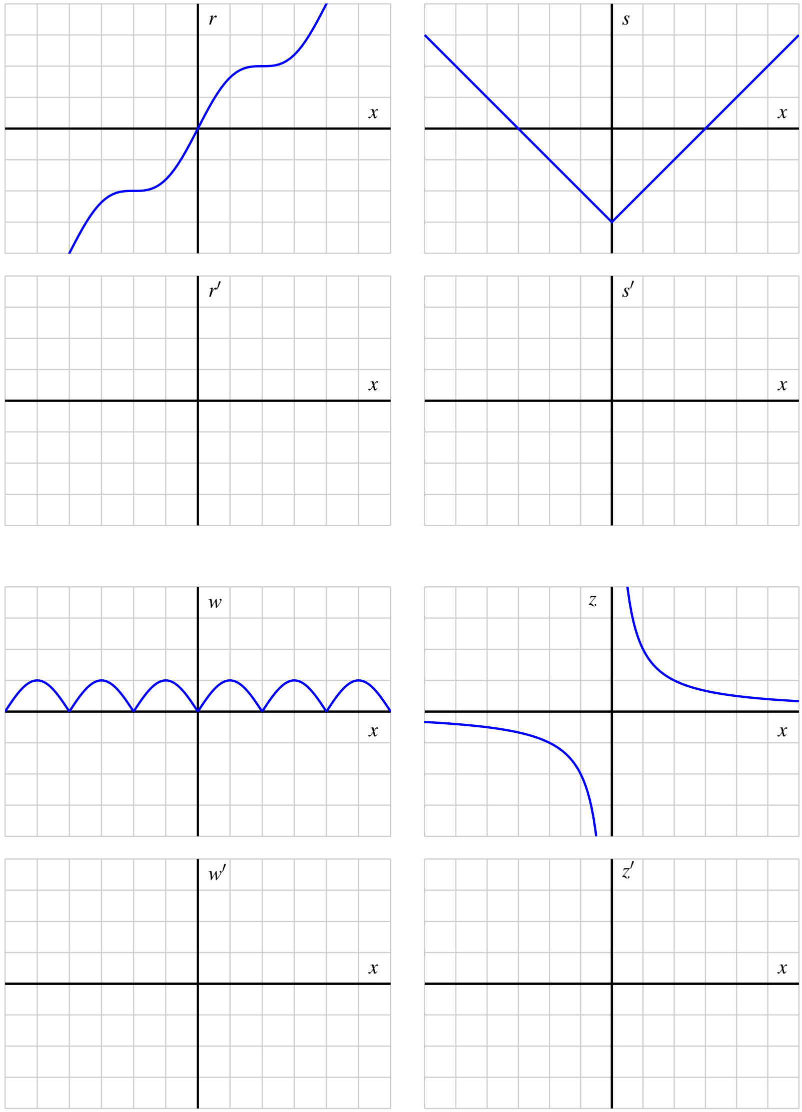
(c) Use the limit definition of the derivative to show that $g'(0) = \lim_{h \to 0} \frac{|h|}{h}$.

(d) Explain why $g'(0)$ fails to exist by using small positive and negative values of h.

Part 3.

For each given graph of $y = f(x)$, sketch an approximate graph of its derivative function, $y = f'(x)$, on the axes immediately below. The scale of the grid for the graph of f is 1×1; assume the horizontal scale of the grid for the graph of f' is identical to that for f. If necessary, adjust and label the vertical scale on the axes for f'.





When you are finished with all 8 graphs, write several sentences that describe your overall process for sketching the graph of the derivative function, given the graph the original function. What are the values of the derivative function that you tend to identify first? What do you do thereafter? How do key traits of the graph of the derivative function exemplify properties of the graph of the original function?

Part 4.

This activity builds on our experience and understanding of how to sketch the graph of f' given the graph of f.

Given the respective graphs of two different functions f, sketch the corresponding graph of f' on the first axes below, and then sketch f'' on the second set of axes. In addition, for each, write several careful sentences that connect the behaviors of f, f', and f''.

Throughout, view the scale of the grid for the graph of f as being 1×1, and assume the horizontal scale of the grid for the graph of f' is identical to that for f. If you need to adjust the vertical scale on the axes for the graph of f' or f'', you should label that accordingly.

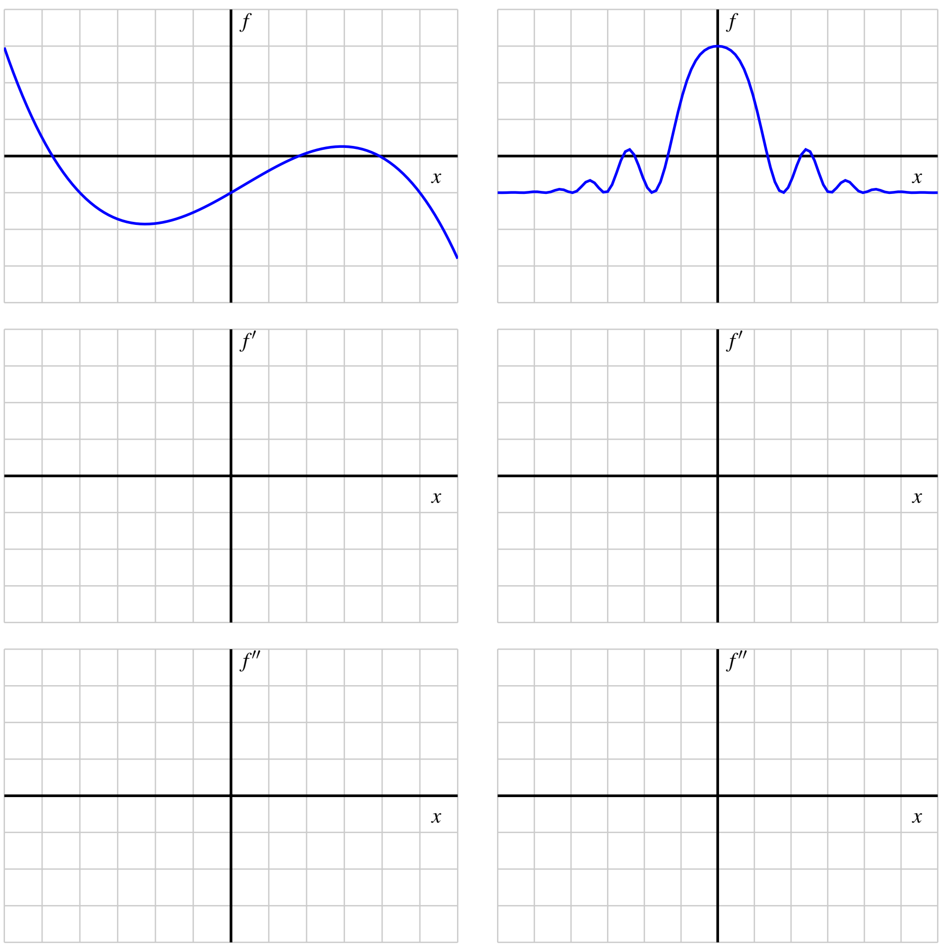


Figure: Two given functions f, with axes provided for plotting f' and f'' below.

Part 5.

Consider the graph of the function $y = p(x)$ that is provided in Figure below. Assume that each portion of the graph of p is a straight line, as pictured.

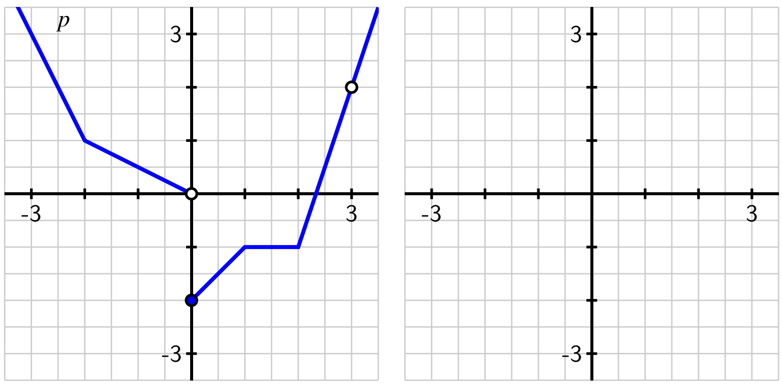


Figure: At left, the piecewise linear function $y = p(x)$. At right, axes for plotting $y = p'(x)$.

(a) State all values of f or which $\lim_{x \to a} (x)$ does not exist.

(b) State all values of a for which p is not continuous at a.

(c) State all values of a for which p is not differentiable at $x = a$.

(d) On the axes provided in Figure, sketch an accurate graph of $y = f'(x)$.