Name______________________________

Activity - Improper Integrals

Part 1.

In this activity we explore the improper integrals $\int_1^\infty \frac{1}{x}\,dx$ and $\int_1^\infty \frac{1}{x^{3/2}}\,dx$.

(a) First we investigate $\int_1^\infty \frac{1}{x}\,dx$.

i. Use the First FTC to determine the exact values of $\int_1^{10} \frac{1}{x}\,dx$, $\int_1^{1000} \frac{1}{x}\,dx$, and $\int_1^{100000} \frac{1}{x}\,dx$. Then, use your calculator to compute a decimal approximation of each result.

ii. Use the First FTC to evaluate the definite integral $\int_1^b \frac{1}{x}\,dx$ (which results in an expression that depends on b).

iii. Now, use your work from (ii.) to evaluate the limit given by

$$\lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx.$$

(b) Next, we investigate $\int_1^\infty \frac{1}{x^{3/2}}\,dx$.

i. Use the First FTC to determine the exact values of $\int_1^{10} \frac{1}{x^{3/2}}\,dx$, $\int_1^{1000} \frac{1}{x^{3/2}}\,dx$, and $\int_1^{100000} \frac{1}{x^{3/2}}\,dx$. Then, use your calculator to compute a decimal approximation of each result.

ii. Use the First FTC to evaluate the definite integral $\int_1^b \frac{1}{x^{3/2}}\,dx$ (which results in an expression that depends on b).

iii. Now, use your work from (ii.) to evaluate the limit given by

$$\lim_{b\to\infty} \int_1^b \frac{1}{x^{3/2}}\,dx.$$

(c) Plot the functions $y = \frac{1}{x}$ and $y = \frac{1}{x^{3/2}}$ on the same coordinate axes for the values $x = 0\ldots10$. How would you compare their behavior as x increases without bound? What is similar? What is different?

(d) How would you characterize the value of $\int_1^\infty \frac{1}{x}\,dx$? of $\int_1^\infty \frac{1}{x^{3/2}}\,dx$? What does this tell us about the respective areas bounded by these two curves for $x \geq 1$?

Part 2.

Determine whether each of the following improper integrals converges or diverges. For each integral that converges, find its exact value.

(a) $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$

(b) $\displaystyle\int_0^\infty e^{-x/4}\,dx$

(c) $\displaystyle\int_2^\infty \frac{9}{(x+5)^{2/3}}\,dx$

(d) $\displaystyle\int_4^\infty \frac{3}{(x+2)^{5/4}}\,dx$

(e) $\displaystyle\int_0^\infty xe^{-x/4}\,dx$

(f) $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$, where p is a positive real number

Part 3.

For each of the following definite integrals, decide whether the integral is improper or not. If the integral is proper, evaluate it using the First FTC. If the integral is improper, determine whether or not the integral converges or diverges; if the integral converges, find its exact value.

(a) $\int_0^1 \frac{1}{x^{1/3}}\,dx$

(b) $\int_0^2 e^{-x}\,dx$

(c) $\int_1^4 \frac{1}{\sqrt{4-x}}\,dx$

(d) $\int_{-2}^2 \frac{1}{x^2}\,dx$

(e) $\int_0^{\pi/2} \tan(x)\,dx$

(f) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$