Name______________________________

Activity - Implicit Differentiation

Part 1. Let f be a differentiable function of x (whose formula is not known) and recall that $\frac{d}{dx}[f(x)]$ and $f'(x)$ are interchangeable notations. Determine each of the following derivatives of combinations of explicit functions of x, the unknown function f, and an arbitrary constant c.

(a) $\frac{d}{dx}\left[x^2 + f(x)\right]$

(b) $\frac{d}{dx}\left[x^2 f(x)\right]$

(c) $\frac{d}{dx}\left[c + x + f(x)^2\right]$

(d) $\frac{d}{dx}\left[f(x^2)\right]$

(e) $\frac{d}{dx}\left[x f(x) + f(cx) + c f(x)\right]$

Part 2.

Consider the curve defined by the equation $x = y^5 - 5y^3 + 4y$, whose graph is pictured below.

(a) Explain why it is not possible to express y as an explicit function of x.

(b) Use implicit differentiation to find a formula for dy/dx.

(c) Use your result from part (b) to find an equation of the line tangent to the graph of $x = y^5 - 5y^3 + 4y$ at the point $(0,1)$.

(d) Use your result from part (b) to determine all of the points at which the graph of $x = y^5 - 5y^3 + 4y$ has a vertical tangent line.

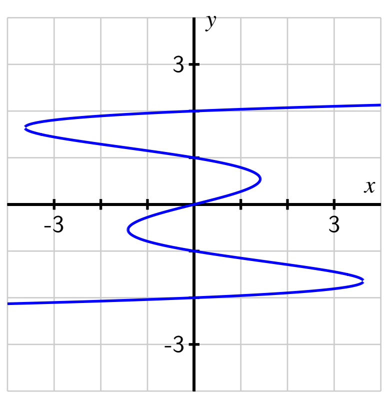

Figure: The curve $x = y^5 - 5y^3 + 4y$.

Part 3. Find dy/dx

(a) $x^3 + y^3 = \tan(y)$

(b) $y^3\sin(y) = x^2 y$

(c) $3xe^{-xy} = y^2$

Part 4.

For each of the following curves, use implicit differentiation to find dy/dx and determine **the equation of the tangent line at the given point**.

(a) $x^3 - y^3 = 6xy, \quad (-3, 3)$

(b) $\sin(y) + y = x^3 + x, \quad (0, 0)$